# VISCOUS FLUID FLOW THROUGH A PIPE WITH ORIFICES

V. A. Kaminskii and É. G. Sinaiskii UDC 532.542

An approximate solution is obtained to the problem of fluid flow through a pipe with orifices, in close agreement with the solution obtained numerically.

An analytical solution has been obtained in [1] to the hydraulic problem concerning the steady flow of a viscous incompressible fluid through a pipe with orifices and a stop at one end, the orifices assumed there to be distributed evenly with a constant total area per unit pipe length. The exact solution to this problem is unwieldy and not suitable for practical design calculations, however, as evidenced by the fact that the author offers a solution obtained numerically as an alternative. In this article here we propose a method of finding an approximate solution to the problem, one which agrees closely with the solution obtained numerically.

With a square-law drag, the flow equations for a viscous fluid are [2]:

$$\left(\varkappa v^2 + \frac{p}{\rho}\right)' = -\frac{\lambda v^2}{2}, \quad v' = -\frac{\sigma}{L}\sqrt{\frac{2p}{\rho}}. \tag{1}$$

At the pipe entrance one may stipulate either the pressure or the velocity. For the purpose of comparison with the results in [1], we will stipulate the entrance pressure. In this case the boundary conditions become

$$p(0) = p_0, \quad v(L) = 0. \tag{2}$$

Changing in (1)-(2) to dimensionless variable, we obtain

$$(2\varkappa u^2 + \varphi)' = -\psi u^2, \quad u' = -\sigma\sqrt{\varphi}, \tag{3}$$

$$\varphi(0) = 1, \quad u(1) = 0. \tag{4}$$

We seek an approximate solution to (3)-(4) in the form

$$u = \sum_{i=1}^{n} a_i (1-z)^i, \tag{5}$$

$$\varphi = \frac{1}{\sigma^2}\left(\sum_{i=1}^{n} i a_i (1-z)^{i-1}\right)^2. \tag{6}$$

Such a choice of u and $\varphi$ satisfies the second of Eqs. (3) and the second of conditions (4). Further using the first of conditions (4), we obtain

$$\frac{1}{\sigma}\sum_{i=1}^{n} i a_i = 1. \tag{7}$$

Letting z = 1 in the first of Eqs. (3), we obtain

$$\varphi'(1) = 0. \tag{8}$$

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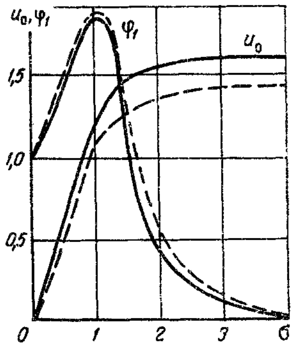


Fig. 1. Relations $u_0(\sigma)$ and $\varphi_1(\sigma)$.

The additional n − 2 conditions needed for determining the coefficients $a_i$ will be found from the requirement that u and $\varphi$ satisfy not the first of Eqs. (3), but the integral relations derived from (3) by integrating it with respect to z from 0 to 1, after both sides have been multiplied by $z^k$:

$$\int_0^1 z^k (2\varkappa u^2 + \varphi)'\, dz = -\psi \int_0^1 u^2 z^k\, dz \quad (k = 0,\ 1,\ \ldots,\ n-3). \tag{9}$$

It is to be noted that this method is analogous to the Karman–Polhausen method in their boundary-layer theory [3].

With the left-hand side of (9) integrated by parts, the second of Eqs. (3) and the boundary conditions (4) yield the following relations:

$$\begin{gathered}\psi \int_0^1 u^2 dz = -\varphi(1) + 1 + 2\varkappa u^2(0),\\ \int_0^1 u\left[(2\varkappa k - \psi z)\, u + \frac{2\sigma'}{\sigma^3}\right] dz = \varphi(1) + \frac{u(0)}{\sigma^2(0)} \quad (k = 1),\\ \int_0^1 u\left[\frac{2kz}{\sigma^3}\sigma' + u\,(2\varkappa k - \psi z)\, z - \frac{k(k-1)}{\sigma^2}\right] z^{k-2} dz = \varphi(1) \quad (2 \leqslant k \leqslant n-3).\end{gathered} \tag{10}$$

Equations (7), (8), and (10) are sufficient for determining n coefficients $a_i$.

Let us complete the calculations for the specific case n = 3 and $\sigma$ = const. For conditions (3) and (8) follows here that

$$a_1 = \sigma - 3a_3 - 2a_2, \quad a_2 = 0. \tag{11}$$

Using then the first of Eqs. (10), we find

$$a_3 = 0{,}5\left\{\left(\frac{6}{\sigma} + \frac{8\sigma\psi}{5} - 8\varkappa\sigma\right) - \left[\left(\frac{6}{\sigma} + \frac{8\sigma\psi}{5} - 8\varkappa\sigma\right)^2 - 4\left(\frac{9}{\sigma^2} - 8\varkappa + \frac{58\psi}{35}\right)\left(\frac{\psi\sigma^2}{3} - 2\varkappa\sigma^2\right)\right]^{1/2}\right\}\left(\frac{9}{\sigma^2} - 8\varkappa + \frac{58\psi}{35}\right)^{-1}. \tag{12}$$

When $\sigma \ll 1$, we have the following expressions:

$$a_3 \approx -\frac{\sigma^3}{3}\left(\varkappa - \frac{\psi}{6}\right), \quad u_0 = u(0) \approx \sigma + \frac{2}{3}\varkappa\sigma^2,$$
$$\varphi_1 = \varphi(1) \approx 1 + 2\sigma^2\left(\varkappa - \frac{\psi}{6}\right).$$

Curves of $u_0$ and $\varphi_1$ as functions of $\sigma$ have been plotted in Fig. 1 (solid lines) according to formulas

$$\begin{gathered} u = a_3(1-z)^3 + (\sigma - 3a_3)(1-z),\\ \varphi = \frac{1}{\sigma^2}(\sigma + 3a_3 z^2 - 6a_3 z)^2,\end{gathered} \tag{13}$$

with $a_3$ determined from (12) for $\sigma/\psi = 0.256$.

A comparison between these curves and analogous ones in Fig. 2 of [1], after obvious corrections have been made in the latter ($\varphi_1$ curve begins not at 0 but at 1), and a comparison between u and $\varphi$ profiles for various values of $\sigma$ indicate that formulas (12) and (13) yield results close to those obtained numerically.

## NOTATION

$\rho$ is the density of fluid in a pipe;
$\varkappa$ is the Coriolis coefficient;

$\psi$, $\lambda$ are the total and the referred coefficient of drag in a pipe;

$\sigma$ is the ratio of total orifice area in pipe wall to inside cross section of a pipe;

$x$ is the longitudinal coordinate ($0 \leq x \leq L$);

$v = v(x)$ is the mean axial velocity of fluid in a pipe;

$p = p(x)$ is the gage pressure (above outside pressure) in a pipe;

$p_0 = p(0)$;

$z = x/L$,

$u = v\sqrt{\rho/2p_0}$,

$\varphi = p/p_0$ are the dimensionless longitudinal coordinate, mean axial velocity, and gage pressure in a pipe.

## LITERATURE CITED


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